ex/ $f(x,y) = x^3 - y^2 - 12x + 6y + 5$

Find all crit points

& ~~what they are~~ classify them

$\frac{\partial f}{\partial x} = 3x^2 - 12$ $\qquad 3x^2 - 12 = 0, \quad 3x^2 = 12, \quad x^2 = 4$

$\boxed{x = 2} \; \boxed{x = -2}$

$\frac{\partial f}{\partial y} = -2y + 6$ $\qquad -2y + 6 = 0 \quad 2y = 6 \quad \boxed{y = 3}$

critical points at $(2,3)$
$(-2,3)$

$\frac{\partial^2 f}{\partial x^2} = 6x$

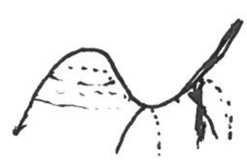
$\frac{\partial^2 f}{\partial y^2} = -2$ $\qquad D(x,y) = 6x(-2)$

$\frac{\partial^2 f}{\partial x \partial y} = 0$

$(2,3) \rightarrow D(2,3) = 6 \cdot 2 \cdot -2$ < 0 ← saddle point

$(-2,3) \rightarrow D(-2,3) = 6(-2) \cdot (-2) > 0$ so either max or min

$\frac{\partial^2 f}{\partial x^2} = (6)(-2) < 0 \rightarrow$ max



ex Find and classifly all critical points:

$$f(x,y) = 2x^2 - x^4 - y^2$$

$f_x = 4x - 4x^3$

$f_y = -2y$

$f_{xx} = 4 - 12x^2$

$f_{yy} = -2$

$f_{xy} = 0$

$$\begin{cases} 0 = 4x - 4x^3 \\ 0 = -2y \end{cases}$$

$$\boxed{y = 0}$$

$0 = 4x - 4x^3$

$0 = x(4 - 4x^2)$

$0 = 4x(1 - x^2)$

$0 = 4x(1-x)(1+x)$ roots at

$x = 0, -1, 1$

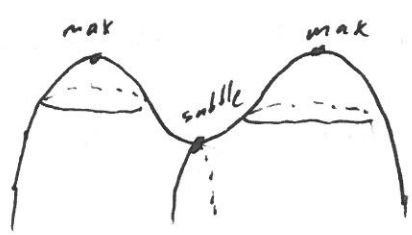
critical points at

$(0,0)$, $(-1,0)$, $(1,0)$

$D(x,y) = (4 - 12x^2)(-2)$

$D(x,y) = -8 + 24x^2$ $\qquad f_{xx} = 4 - 12x^2$

$$\begin{cases} D(0,0) = -8 & f_{xx}(0,0) = 4 & \text{saddles} \\ D(-1,0) = 16 & f_{xx}(-1,0) = -8 & \text{max} \\ D(1,0) = 16 & f_{xx}(1,0) = -8 & \text{max} \end{cases}$$



Ex/ A firm sells ~~produ~~ a product to two different countries. They charge different amounts in each country.

x: units sold in 1st country

y: units sold in 2nd country.

Using supply and demand laws they set the prices as

$97 - (x/10)$ in the first country and

$83 - (y/20)$ in the second.

The cost of production is $20{,}000 + 3(x+y)$

- what values of x and y maximize profit?

Profit = revenue − costs

revenue = units sold × price

So $f(x,y)$ = revenue from 1st country + revenue from 2nd country − cost

rev from first $= \left(97 - \frac{x}{10}\right)x$

rev from 2nd $= \left(83 - \frac{y}{20}\right)y$

$$\begin{cases} 4x - 2y = 6 \\ 2x = 10y \end{cases} \Rightarrow \begin{cases} 4x - 2y = 6 \\ x = 5y \end{cases}$$

substitute $x = 5y$ to 1st eq.

$$4(5y) - 2y = 6$$

$$\Rightarrow 20y - 2y = 6$$

$$\Rightarrow 18y = 6$$

$\boxed{y = \frac{1}{3}}$ since $x = 5y$

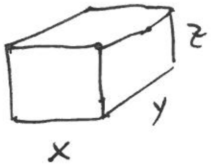
$\boxed{x = \frac{5}{3}}$

minimum at $\left(\frac{5}{3}, \frac{1}{3}\right)$

what is the value?

$$f\left(\frac{5}{3}, \frac{1}{3}\right) = 2\left(\frac{5}{3}\right)^2 - 2\left(\frac{5}{3}\right)\left(\frac{1}{3}\right) + 5\left(\frac{1}{3}\right)^2 - 6\left(\frac{5}{3}\right) + 5 = 0$$

Find the dimensions of a rectangular box that minimizes surface area and has a volume of 1,000 cubic inches



Volume: $xyz = 1000$

Surface Area $= \cancel{4}\ 2xy + 2xz + 2yz$

$$z = \frac{1000}{xy}$$

$$SA = f(x,y) = 2xy + 2000y^{-1} + 2000x^{-1}$$

$$f_x = 2y - 2000x^{-2} = 0 \implies y = 1000x^{-2}$$

$$f_y = 2x - 2000y^{-2} = 0$$

$$0 = 2x - 2000(1000x^{-2})^{-2}$$

$$0 = 2x - 2000(1000)^{-2}x^4$$

$$0 = x - 1000(1000)^{-2}x^4$$

$$0 = x - \frac{x^4}{1000}$$

$$0 = 1000x - x^4$$

$$0 = x(1000 - x^3)$$

$$\boxed{x = 0} \quad \text{or} \quad \boxed{x = 10}$$

$$\downarrow$$

$$y = \frac{1000}{100} \implies \boxed{y = 10}$$

$$1000 = xyz$$

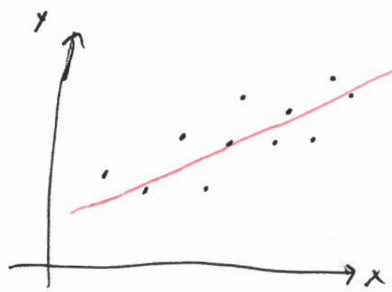
$$1000 = (10)(10)z \implies \boxed{z = 10}$$

7.5 The Method of Least Squares

Why do we collect data?

One reason: to make predictions

Given some data $(x_1, y_1), (x_2, y_2), \ldots, (x_N, y_N)$

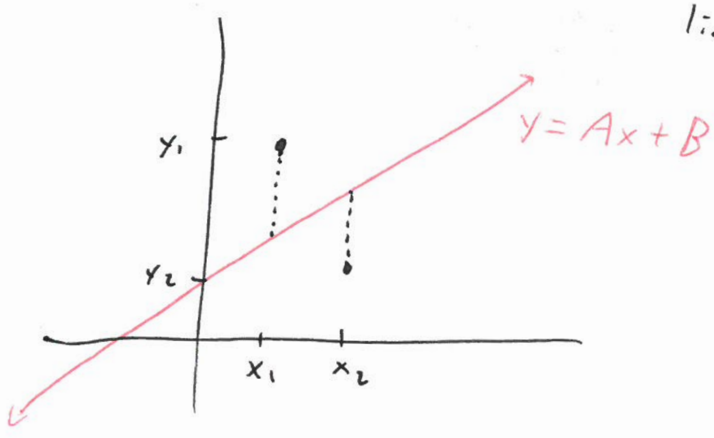


we want to find a straight line that best fits these points.

In the real world not all data will lie on a line, there will be some error.

How do we measure error?

Vertical distance from the line



Error for (x_1, y_1)

$$E_1 = Ax_1 + B - y_1$$

for (x_2, y_2)

$$E_2 = Ax_2 + B - y_2$$

Here E_1 is negative

E_2 is positive

To add up all error so nothing "cancels out"

look at square of error

E_1^2 is positive

E_2^2 is positive

Total error is $E = E_1^2 + E_2^2 + \dots + E_N^2$

if we have N points.

Our goal: <u>minimize</u> error.

Ex How much Error is there for the points $(1,3), (2,6), (3,8), (4,6)$ fitted with the line $y = 1.1x + 3$?

	x	$y = 1.1x + 3$	E_i
$(1,3)$	1	4.1	$4.1 - 3 = 1.1$
$(2,6)$	2	5.2	$5.2 - 6 = -0.8$
$(3,8)$	3	6.3	$6.3 - 8 = -1.7$
$(4,6)$	4	7.4	$7.4 - 6 = 1.4$

$$E = 1.1^2 + (-0.8)^2 + (-1.7)^2 + (1.4)^2$$

$$\cancel{= 6.064}$$

$$= 6.7$$

How do we minimize error?

Can we use a different line?

$$y = Ax + B$$

Error only depends on A and B

can use optimization with partial derivatives.